Spectrum sensing of OFDM signals in frequency domain using histogram based ratio test

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***Abstract*— The paper introduces an OFDM signal detector based on cyclic correlation function. The detector belongs to the class of blind SCD detectors. The paper presents the main relations connected with the idea of cyclostationarity and cyclostationary features of OFDM signals. Results of simulations are presented as well.**

***Keywords*— hybrid detectors architecture, cyclostationary feature detection, cyclic autocorrelation function, OFDM**

I. Introduction

Sensing of the primary user's (PU) signals is one of the most important features for cognitive radio (CR) operations. For OFDM signals apart from blind methods like energy detection (ED) [1] also other methods based on the use of either cyclic prefix (CP) or pilot tones are proposed. Sensing algorithm based on cross-correlation connected with pilot tones was considered in [2]. Since CP is a common feature of almost all OFDM systems numerous algorithms utilize it. Generalized likelihood ratio test (GLRT) based on CP presence was proposed in [3, 4]. Presence of CP in a received signal is revealed in the form of cyclic peaks of the autocorrelation function. This feature is exploited by cyclostationary algorithms [5, 6] which seem to outperform the other ones.

In this letter a frequency domain approach is proposed, which is based on an empirical probability density function (epdf) estimated by an histogram. The major contributions are as follows:

1) Periodicity of an OFDM signal similar revealed in time-domain by cyclic autocorrelation function (CAF) can be obtained in frequency-domain by testing the ratio between cardinality of the bins of an adequate histogram.

2) In this way a new frequency origin decision metric is introduced.

3) Proposed histogram based ratio test (HBRT) carried out on the OFDM based WiMAX signal model outperforms CAF based algorithms.

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II. CYCLIC AUTOCORRELATION

According to [5], the complex x(t) process with the average zero value is cyclostationary in a wide sense, if its autocorrelation function is periodic with repetition period T_f and can be represented as a Fourier series

$$R_{xx}(t,\tau) = \sum_{\alpha} R_{xx}^{\alpha}(\tau) e^{j2\pi\alpha t} \tag{1}$$

$\alpha = k/T_f$, k = 1,2,3....

The Fourier series coefficients depending on the time lag have the following form

$$R_{xx}^{\alpha}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} R_{xx}(t,\tau) e^{-j2\pi\alpha t} dt \tag{2}$$

This function is called CAF [7] and its peaks – connected with specific lags (Fig 1) – are used for detection of the presence of the OFDM signal.

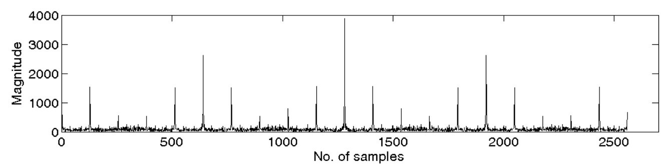

Fig. 1. Periodicity of autocorrelation for 4 WiMAX OFDM symbols, lag expressed as a number of samples, CP length 1/4.

III. HISTOGRAM BASED RATIO TEST

In this correspondence we propose another approach which reveals features connected with the presence of OFDM subcarriers but in the frequency domain.

Let us consider a sensing problem as a hypothesis test

$$\begin{aligned} H_0 &: X_i = W_i \\ H_1 &: X_i = S_i + W_i \end{aligned} \tag{3}$$

where S_i – complex samples of PU's signal, W_i – independent identically distributed (iid) complex Gaussian noise. Now we consider variable Y_i = FFT(X_i). Since Fast Fourier Transformation (FFT) denotes basically the multiplication of input samples by (cosϕ - j sinϕ), so 2-dimensional pdf of normal variables (X,Y), $f(x,y)$ transformed to random variables X=R cosΦ, Y=R sinΦ has pdf $f(r,\phi)=f_r(r)f_\phi(\phi)$ where $f_r(r)$ has Rayleigh distribution and $f_\phi(\phi)$ has uniform distribution. Thus, for the hypothesis H_0 random variable $|Y_i$ = FFT(W_i)$|$ is independent and Rayleigh distributed. Hence

$$Z_i = \left| Y_i Y^*_{i+n} \right|, \quad n \neq 0 \tag{4}$$

is identically gamma distributed for any lag n different to zero, Fig. 2.

It is worth to note that the histograms presented in Fig. 2 are estimates of the pdfs. To construct them the range of values is divided into a series of non-overlapping intervals (bins). The value of each bin reflects the number of samples falling into each interval. Assuming that Z_i is identically distributed, the ratio between arbitrarily chosen bins also shall be more or less constant for any lag n different from zero.

On the other hand, if X_i contains some samples of deterministic signals it should reveal it for some lag n as a distinctive increase in the ratio of histogram bins of Z_i (Fig 2c). This leads us to the proposed sensing algorithm.

Proposed sensing method: The spectrum sensing problem is formulated as

$$\begin{aligned} H_0 &: A \le \lambda \\ H_1 &: A > \lambda \end{aligned} \tag{5}$$

where λ is a threshold and A stands for the ratio between cardinality of the first bin of the histogram and sum of all other bins for arbitrarily chosen lag.

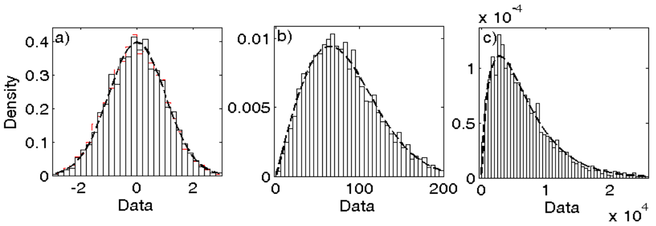

Fig. 2. Histogram and pdf a) normal distribution of real and imaginary parts of W_i, b) Rayleigh distribution of $|Y_i$=FFT(W_i)$|$, c) gamma distribution of $|Z_i|$.

$$A = \frac{A_1}{\sum_{i=2}^{N} A_i} \tag{6}$$

Fig. 3 shows A ratio as a function of lag for two cases: OFDM signal with noise and noise with variances corresponding to SNR in the previous case. As it can be seen the presence of PU's signal is manifested as periodic peaks for some lags.

The proposed method can be summarized in the following steps:

Step 1: Make FFT of the complex samples X_i of the received signal.

Step 2: Calculate Z_i according to (4) for specific lag n where the peak of A is expected.

Step 3: Calculate the histogram upon modulus of Z_i.

Step 4: Calculate the test statistics A according to (6).

Step 5: Find the threshold λ for given probability of false alarm.

Step 6: Accept the null hypothesis H_0 if $A \le \lambda$.

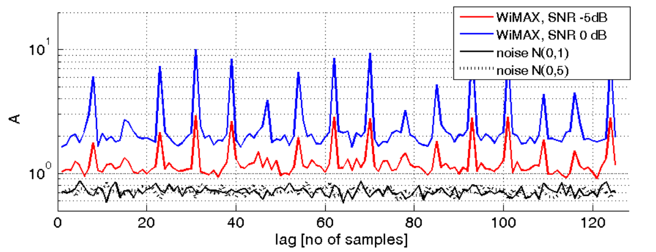

Fig. 3. Histogram's bins ratio for WiMAX OFDM signal, CP 1/32 and Gaussian noise with variances corresponding to SNR.

It should be stressed that to calculate *Step 2* some a'priori knowledge about signal structure is necessary. In the CAF case positions where peaks are expected depends on symbols duration, LCP size as well as sampling rate at SU side. In the HBRT case position of peaks depends on the ratio between FFT size used in the sensing algorithm and FFT size of PU OFDM system as well as sampling rate of SU system. Note, that change in sampling rate will cause change in peak positions (expressed as a number of samples). Assuming perfect timing without oversampling peaks of A for WiMAX system are present every Δ samples:

$$n = m \cdot \Delta = 2^{k+2}, m = 1, 2, 3... \tag{7}$$

where k stands for FFT ratio expressed as:

$$\frac{FFT_{HBRT}}{FFT_{WiMAX}} = 2^k \tag{8}$$

IV. SIMULATION RESULTS

To show the performance of the proposed algorithm WiMAX signal model was chosen. Sensing time equal to 0.512 ms and 4.1 ms was set according to tested FFT size (4096 or 32768) and was equal to duration of 6.4 and 51.2 OFDM symbols with CP length 1/4, respectively. The results obtained were compared with CAF detector considered in [8]. Details of simulation scenario are summarized in Table 1.

TABLE I. SUMMARY OF THE SIMULATIONS SCENARIO

Tested PU's signal	WiMAX
Length of CP	1/4
No. of OFDM symb.	6.4 symb. ÷ 51.2 symb
Sensing time	0.512 ms ÷ 4.1 ms
SNR	-20dB to 0 dB
threshold λ	obtained for 10 000 Monte Carlo simulations
lag n where the peak of A is expected	512 samples
FFT size	4096 ÷ 32768
No. of histogram's bins	10

In Fig. 4 Probabilities of detection against SNR are plotted for different numbers of samples as well as for two lengths of contention periods (CP). It is worth to note that CP=1/4 is the longest and CP = 1/32 is the shortest for WIMAX systems. Thus, results for given number of samples for these two values of CP creates borders of possible performances of HBRT. To maintain fair conditions for comparison our simulations was made for specific size of samples what resulted with different numbers of symbols, eg. 4096 samples for WiMAX system with CP = 1/4 means that there were transmitted 6.4 OFDM symbols whereas for CP = 1/32 it was 7.75 OFDM symbols.

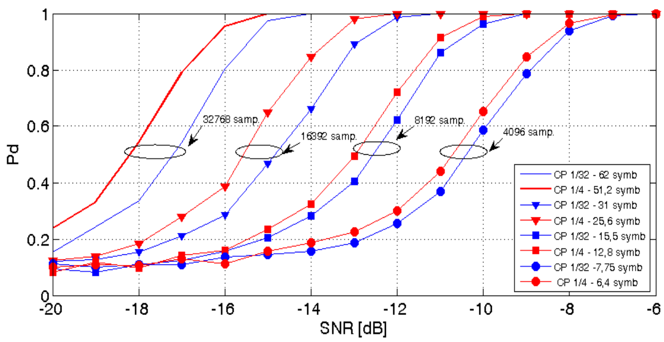

Fig. 4. Detection probability against SNR over AWGN channel with Pf=0.1

In Fig. 5, the values of the detection probability against SNR are plotted for two sensing methods and two sensing times. The probability of false alarm was set to 0.1. It can be seen that the proposed HBRT detector achieved probability of detection at the level of 0.9 at SNR = -16.5 dB for the case of 51.2 symbols, whereas CAF detector achieved this probability at SNR = - 13 dB. For the detection probability ≥ 0.9 HBRT detector has better performance than CAF, also for 0.512 ms/4096 FFT case but in a limited range.

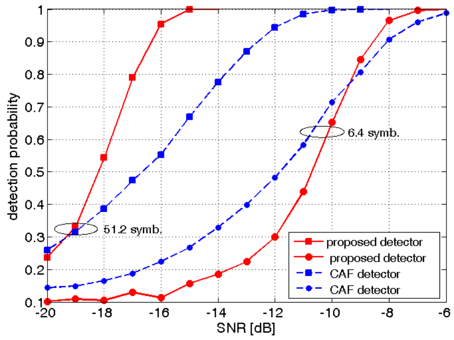

Fig. 5. Comparison of detection probability against SNR over AWGN channel with Pf=0.1 for CAF and HBRT detectors

Results presented in Fig 4 and Fig. 5 were obtained for probability of false alarm equal to 0.1 and for threshold values came from Monte Carlo simulations, Fig 6.

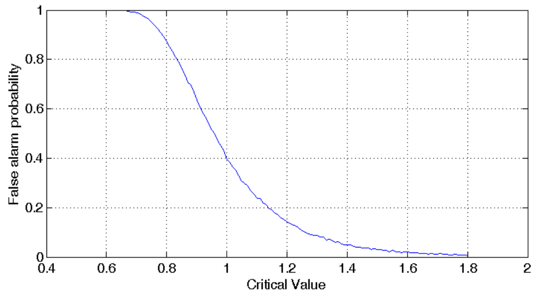

Fig. 6. False alarm probability in a threshold function

For SNR lower than -9 dB CAF based detector achieved better results. To analyze this phenomenon probability of detection against probability of false alarm are plotted in Fig. 5 for two cases: SNR = -16dB for 51.2 symbols (Fig. 7a) and SNR=-10 dB for 6.4 symbols (Fig. 7 b). From Fig. 5 it can be seen that detection probability goes to 1 much faster for the proposed HBRT detector compared to CAF detector. But, if the number of samples is not sufficient (Fig. 7b) at some SNR, this superiority turns in favor to CAF detector. These results showed that taking into account sensitivity of detection expressed as SNR level for certain cognitive system parameters of HBRT detector shall be tuned.

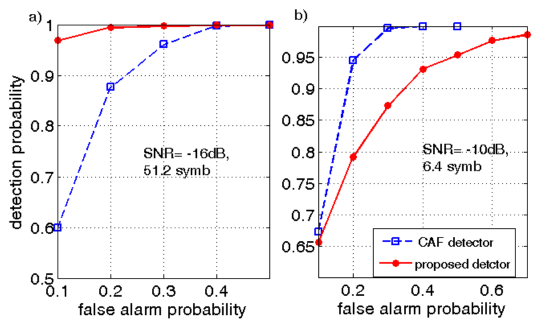

Fig. 7. Detection probability against false alarm probability over AWGN channel with SNR = -16dB for 51.2 symbols and SNR=-10dB for 6.4 symbols.

As a last step performance of HBRT detector was tested in a fading environment. As a fading environment ITU-R path loss model was chosen [9]. The recommendation M.1225 specifies three different test environments: indoor office, outdoor-to-indoor pedestrian, and vehicular – high antenna. Since the delay spread can vary significantly there are specified two different delay spreads for each test environment: low delay spread (A), and medium delay spread (B). For the test four profiles were chosen Pedestrian A, B and Vehicular A, B. Apart from that another necessary parameter is maximum Doppler spread. This parameter was chosen according to [10]. Summary of fading environments is presented in Table 2.

TABLE II. RAYLEIGH FADING MULTIPATH ENVIRONOMENT

Carrier Freq. [GHz]	Speed [km/h]	Max Doppler Spread [Hz]	Coherence Time [ms]	ITU-R profile
2.5	2	4.6	200	Pedestrian A
				Pedestrian B
	100	231.5	4	Vehicular A
				Vehicular B

Obtained results are presented in Fig. 8 and for detection probability equal to 0.9 summarized in Table 3. Δ stands for the difference between AWGN and fading performance.

As it can be seen for some fading profiles HBRT achieved even slightly better performance than for AWGN environment. However, excluding outstanding results (HBRT – Pedestrian B, and CAF – Pedestrian A) the rest of the results are merely similar and can be summarized as a thumb rule: fading effects can deteriorate performance of both algorithm at the level of ~2 dB.

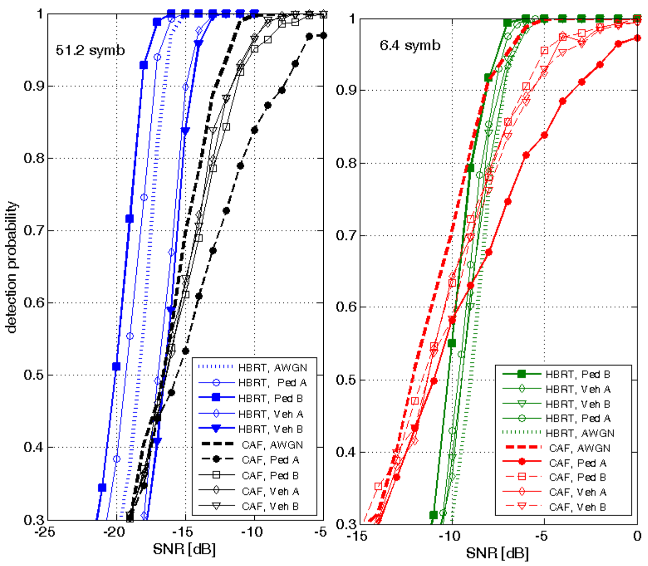

Fig. 8. Detection probability against SNR over fading channels

TABLE III. COMPARISON OF DETECTION PROBABILITY PD=0.9 AGAINST SNR OVER FADING CHANNELS

Pd=09	6,4 symb. HBRT [dB]	Δ [dB]	CAF [dB]	Δ [dB]	51,2 symb. HBRT [dB]	Δ [dB]	CAF [dB]	Δ [dB]
AWGN	-7.1	---	-8.1	---	-16.5	---	-12.7	---
Ped. A	-7.7	**0.5**	-3.5	***-4.6***	-17.2	**0.7**	-7.8	***-4.9***
Ped. B	-8.1	***1.0***	-6.1	**-2.0**	-18.1	***1.6***	-11.2	**-1.5**
Veh. A	-7.2	**0**	-5.7	**-2.4**	-15.0	**-1.5**	-11.6	**-1.1**
Veh. B	-7.4	**0.3**	-5.7	**-2.4**	-14.5	**-2.0**	-11.6	**-1.1**

V. CONCLUSION

The proposed detector can outperform detectors based on CAF for sensing time equal to duration of ~ 50 OFDM WiMAX symbols. For considered in the literature sensing time (up to 100 ms) we can expected even better results. But for sensing time equal to ~ 6 symbols the results were worse than for CAF detector. One of the possible explanations is not optimal FFT size and number of histogram's bins. Our future work will focus on these two points of freedom as well as the theoretical probability of false alarm.

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